

Two surface multipactor discharge with two-frequency rf fields and space-charge effects

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Asif Iqbal,¹ John Verboncoeur,^{1,2} and Peng Zhang^{1,a)}

AFFILIATIONS

¹Department of Electrical and Computer Engineering, Michigan State University, East Lansing, Michigan 48824, USA

²Department of Computational Mathematics, Science, and Engineering, Michigan State University, East Lansing, Michigan 48824, USA

^{a)}Author to whom correspondence should be addressed: pz@egr.msu.edu

ABSTRACT

This paper presents two-surface multipactor discharge with two-frequency rf fields using Monte Carlo simulations and Computer Simulation Technology (CST) Particle Studio. The effects of the relative strength and phase of the second carrier mode on multipactor susceptibility and time dependent physics are studied. Compared to single-frequency rf operation, shrinkage of multipactor susceptibility regions is observed for different configurations of two-frequency rf operation. The presence of a second carrier mode in the rf field results in mixed multipactor modes in which electrons take a fixed time period to complete a round trip between the two surfaces, while the time for electrons to traverse the gap in each direction is found to be different. CST simulation reveals that the space-charge effect reduces the electron growth rate and causes shrinkage of multipactor susceptibility bands.

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I. INTRODUCTION

Multipactor^{1–7} is a nonlinear ac discharge phenomenon occurring in rf systems. It refers to an electron avalanche driven by a high-frequency rf field that sustains itself by charge growth through secondary electron emission^{8–10} from metallic or dielectric surfaces. While occasionally useful for specific applications such as vacuum conditioning and harmonic generation,^{4,11} multipactor is usually detrimental to rf systems and causes various damages to rf devices, including breakdown of dielectric windows,^{12–14} erosion of metallic structures, melting of internal components, and perforation of vacuum walls.² Multipactor discharge also causes intermodulation of signals¹⁵ and signal quality distortion.¹⁶ Therefore, reducing multipactor is of significant practical interest in the design and operation of high-power microwave (HPM) sources, pulsed power devices, rf accelerators,¹⁴ and space-based communication systems.¹⁷

The mechanisms of multipactor discharge have been extensively studied and are well understood, especially for single-frequency rf operation. However, the growing importance of multicarrier rf operation in modern communication systems as well as in satellite communications^{16,18} in recent years has rendered the understanding of multicarrier rf field-induced multipactor discharge imperative.^{19–21} Different theoretical models have been adopted by researchers in this regard such as nonstationary^{22,23} and quasi-stationary²⁴ statistical theories of Anza *et al.* and the map-based theory^{25,26} of Siddiqi and Kishkek. In addition, Particle-in-Cell (PIC)²⁷ and Monte Carlo^{11,16,28,29}

simulations have been frequently employed as well. Recently, two-frequency-induced multipactor on a single dielectric surface has been studied in both time domain and frequency domain.^{15,19,30–33} Multicarrier rf field-induced two-surface multipactor has also been conducted through numerical^{34,35} and analytical³⁶ methods. However, important aspects of two-surface multipactor discharge due to multi-frequency rf operation, such as multipactor susceptibility and time-dependent physics, have not been adequately studied and are still not well understood.

This paper aims to provide a detailed account of multipactor discharge in parallel plate geometry with two-frequency rf fields. We start with a brief introduction to the electron dynamics and the force law in Sec. II. Section III presents various methods used in our study, including the analytical model, Monte Carlo (MC) simulation, and Computer Simulation Technology (CST) model.³⁷ Results of our study are presented in Sec. IV. In Sec. V, the effects of space-charge^{38–41} on multipactor susceptibility and time-dependent physics are discussed through case studies in CST. A summary with the main conclusions and possible future work is presented in Sec. VI.

II. THE FORCE LAW

Figure 1 depicts a simplified schematic of the two-surface planar multipactor. An rf electric field $E_y = E_{rf} \sin(\omega t + \theta) + \beta E_{rf} \sin(n(\omega t + \theta) + \gamma)$ is applied normal to the parallel plates A and B along the y-direction. Here, E_{rf} is the peak electric field strength, ω is the angular

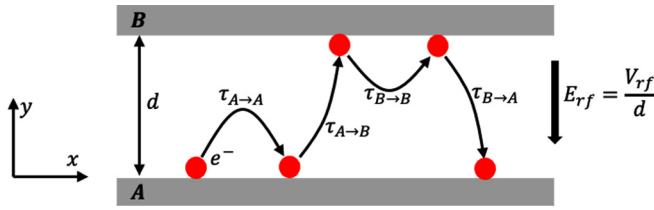


FIG. 1. Multipactor discharge with an electric field oscillating between two metal electrodes A and B separated by a gap distance d .

frequency, and θ is the initial phase of the fundamental carrier mode of the electric field. β is the field strength of the second carrier mode relative to the fundamental mode, γ is the relative phase of the second carrier mode when $(\omega t + \theta) = 0$ or integer multiple of 2π , and n is the ratio of the two carrier frequencies. Assuming one-dimensional particle motion along the y -direction, the force law governing the electron trajectories is as follows:

$$a_y(t) = \frac{eV_{rf}}{md} [\sin(\omega t + \theta) + \beta \sin(n\omega t + \theta + \gamma)], \quad (1)$$

where $a_y(t)$ is the acceleration, V_{rf} is the peak voltage, and d is the gap distance between plates A and B. The velocity of the electron is obtained as

$$v_y(t) = \frac{eV_{rf}}{m\omega d} \left\{ \cos \theta - \cos(\omega t + \theta) + \frac{\beta}{n} [\cos(n\theta + \gamma) - \cos(n\omega t + \theta + \gamma)] \right\} + v_0, \quad (2)$$

and the instantaneous position of a multipactor electron along the y -direction is obtained as

$$y(t) = \frac{eV_{rf}}{m\omega^2 d} \left\{ \omega t \cos \theta - \sin(\omega t + \theta) + \sin \theta + \frac{\beta}{n^2} [n\omega t \cos(n\theta + \gamma) - \sin(n\omega t + \theta + \gamma) + \sin(n\theta + \gamma)] \right\} + v_0 t + y_0. \quad (3)$$

Here, v_0 is the initial velocity and y_0 is the initial position of the electron at $t = 0$.

III. METHODS

A. Analytical model for single-frequency susceptibility boundaries

For single frequency rf operation [$\beta = 0$ in Eqs. (1)–(3)], Vaughan's analytical model² considers only electron motion that traverses the gap, i.e., the trajectories labeled as $\tau_{A \rightarrow B}$ and $\tau_{B \rightarrow A}$ in Fig. 1. The multipactor resonance condition² specifies that an electron emitted from one surface at the initial rf phase θ must traverse the electrode spacing, d , and impact the opposing surface with a time duration of $N\pi/\omega$, where N is a positive odd integer, i.e., $y = d$ at $\omega t = N\pi + \theta$. Invoking this condition in (3), the condition for multipactor resonance in a parallel plate geometry can be easily found in the absence of space charge as²

$$V_{rf} = \frac{m\omega d(\omega d - v_0 N\pi)}{e N\pi \cos \theta + 2\sin \theta}. \quad (4)$$

The lower multipactor boundary is obtained from (4) with $\theta = \arctan(\frac{2}{N\pi})$, which minimizes V_{rf} regardless of the value of emission velocity v_0 . The upper boundary is given by (4) with the maximum negative value of the angle θ , determined by the condition that the initial emission velocity v_0 just allows the electron to escape from the emission surface against the initially retarding field.² The detailed procedure to determine this maximum negative value of θ and, therefore, the upper multipactor boundary is revisited in Sec. 5.1.2 of Ref. 32.

In order to have multipactor growth, the secondary electron yield (SEY), defined as the average number of secondary electrons produced by the impact of primary electrons upon the surface, must be larger than one. The curve of SEY vs electron impact energy is characterized by two crossover points at which SEY is unity and is larger than one in between. By setting the impact energy of electrons when reaching the opposite electrode surface to the first and second crossover incident energies ($V_{1,2}$), we have two more boundaries,⁴²

$$V_{rf}(V_0; V_{1,2}, fd, N) = \frac{m\omega d}{2e} \sqrt{\frac{(N\pi)^2 + 4}{2m} (\sqrt{V_{1,2}} - \sqrt{V_0})^2 + \left(\omega d - N\pi\sqrt{\frac{2V_0}{m}}\right)^2 - N\pi \left(\sqrt{\frac{2V_{1,2}}{m}} - \sqrt{\frac{2V_0}{m}}\right) \left(\omega d - N\pi\sqrt{\frac{2V_0}{m}}\right)}, \quad (5)$$

where $V_0 = mv_0^2/2$ is the electron emission energy.

In this study, we assume copper (Cu) as the electrode material and employ Vaughan's model for secondary electron emission.⁹ We use first crossover incident energy $V_1 = 42$ eV, second crossover incident energy $V_2 = 3054$ eV, peak incident energy $V_{\max 0} = 277.5$ eV, and peak SEY $\delta_{\max 0} = 2.0887$; all these values are for normal incidence. A typical two-surface multipactor susceptibility diagram with single-frequency rf operation is shown in Fig. 2.

For two-frequency rf operation, different combinations of n , β , and γ can lead to very different rf waveshapes. Therefore, it would be questionable to make a general assumption about certain resonant multipactor modes, as electron motion labeled as $\tau_{A \rightarrow A}$ or $\tau_{B \rightarrow B}$ in Fig. 1 may become important, and the motions in $\tau_{A \rightarrow B}$ and $\tau_{B \rightarrow A}$ are typically asymmetric. Thus, in this article, we resort to simulations with Monte Carlo (MC) and CST to study the two-frequency rf field-induced multipactor discharge, while the analytical model for

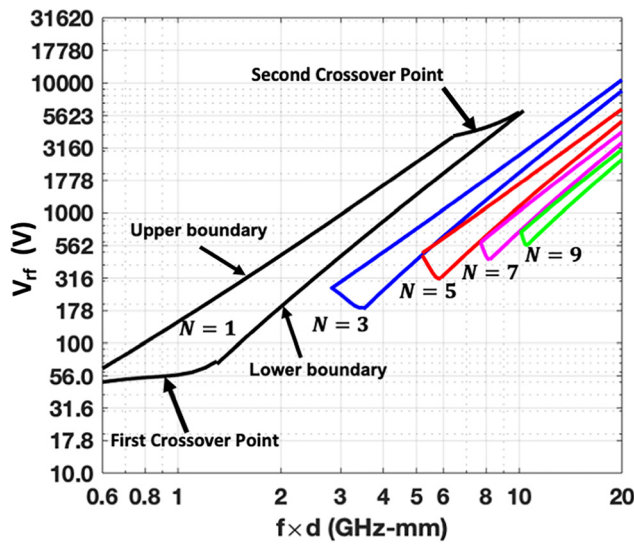


FIG. 2. Two-surface multipactor susceptibility diagram with single-frequency rf operation. Multipactor susceptibility regions for the first five modes $N = 1, 3, 5, 7$, and 9 are shown with emission energy, $V_0 = \frac{1}{2}mv_0^2 = 2$ eV. The boundaries corresponding to the first and second crossover points are calculated from (5).

single-frequency rf operation is primarily used for verifying our MC and CST models.

B. Monte Carlo (MC) simulation

To calculate the growth rate of the multipactor discharge, we follow the trajectory of a weighted macroparticle over a large number of impacts in a MC simulation.^{4,5,43} The initial rf phase, θ , is uniformly distributed over $0 \leq \theta < 2\pi$. Each time a macroparticle leaves the surface of an electrode, we assign it a random initial energy $V_0 = (1/2)mv_0^2$ and angle ϕ (with respect to the surface parallel) according to the following distributions:³

$$f(V_0) = \frac{V_0}{V_{0m}^2} e^{-\left(\frac{V_0}{V_{0m}}\right)}, \quad (6a)$$

$$g(\phi) = \frac{1}{2} \sin \phi, \quad 0 < \phi < \pi, \quad (6b)$$

where V_{0m} is the peak of the distribution of emission energies.³ Referring to Fig. 1, transit times of an electron in flight for single surface impacts on plate A ($\tau_{A \rightarrow A}$) and plate B ($\tau_{B \rightarrow B}$) are calculated by solving (3) for $y_0 = y(\tau_{A \rightarrow A}) = 0$ and $y_0 = y(\tau_{B \rightarrow B}) = d$, respectively. Transit times for two-surface impacts $\tau_{A \rightarrow B}$ and $\tau_{B \rightarrow A}$ are calculated by solving (3) for $y_0 = 0$, $y(\tau_{A \rightarrow B}) = d$ and $y_0 = d$, $y(\tau_{B \rightarrow A}) = 0$, respectively. Then, we calculate the impact energy, $V_i = mv_i^2/2$, from (2) and the secondary electron yield, δ , from Vaughan's model.⁹ We use this value of the yield to adjust the charge and mass on the macroparticle and then emit it again with a random velocity. We repeat the process to obtain a series of yields ($\delta_1, \delta_2, \dots, \delta_{N_i}$) for N_i consecutive impacts. The rf phase θ is calculated self-consistently ($\theta_{i+1} = \theta_i + \omega\tau_i$) at the beginning of each flight for a given macroparticle. The average value of secondary yield over N_i impacts is then calculated as $\bar{\delta} = (\delta_1 \cdot \delta_2 \dots \delta_{N_i})^{1/N_i}$, where $N_i = 20$ is used in the calculation. It is checked that multipactor susceptibility charts remain unchanged if the number of impacts is increased beyond $N_i = 20$ in the MC simulation. For a specific combination of angular rf frequency ω , gap distance d , and accelerating voltage V_{rf} , if we find a valid electron trajectory for any of the initial rf phase θ for which $\bar{\delta} > 1$, we conclude that multipactor discharge develops in the system for this set of parameters.

C. CST model

CST Particle Studio (CST PS)³⁷ is a particle-in-cell (PIC) code that includes secondary electron emission and has been widely used for the simulation of rf vacuum electronics^{44–46} as well as for multipactor discharge.⁴⁷ For the simulation of two-surface multipactor using CST, we consider a simple parallel plate structure as shown in Fig. 3(a). Plates A and B are made of copper with a vacuum gap between them. Each plate has a length $l = 20$ mm, width $w = 20$ mm, and thickness $t_p = 0.2$ mm. A constant amplitude of $f = 1$ GHz excitation is applied

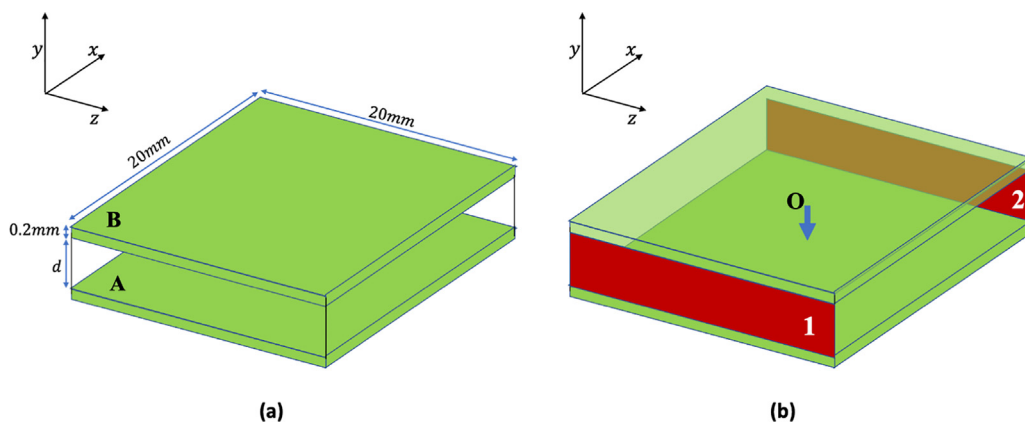


FIG. 3. (a) Schematic of the parallel plate geometry used in the CST simulation for two-surface multipactor. A and B are the bottom and top plates, respectively. The plates are separated by a gap distance d . (b) Waveguide ports 1 and 2 used to apply the 1-GHz excitation shown as red planes. O is the particle source with an emission area of 10^{-4} mm².

to a waveguide port on one end [port 1 in Fig. 3(b)] of the parallel plate structure and exits through a waveguide port on the opposite end [port 2 in Fig. 3(b)]. For two frequency rf operation, the excitation signal is defined as $E_y = \frac{V_0}{d} [\sin(\omega t + \theta) + \beta \sin(n(\omega t + \theta) + \gamma)]$. A variation in fd is achieved in the simulation by varying the gap distance, d .

For secondary electron emission, we use the built-in Vaughan's model in CST, which assumes an emission energy distribution that is gamma distributed and weighted by a temperature T_e .³⁷ For the simulations in this study, we assume a temperature of $T_e = 2$ eV. Secondary electrons are emitted at angles relative to the surface normal with the following probability distribution function (PDF):³⁷

$$f(\theta_N) = \cos\theta_N, \quad \theta_N \in [0, \pi/2]. \quad (7)$$

A particle source O with an emission area of 10^{-4} mm² is located at the center of the top plate pointed toward the $-y$ direction [Fig. 3(b)]. The particle source emits seed particles into the vacuum gap between the plates during the first period ($t_{\text{emission}} = 1$ ns) of the excitation at a constant rate and remains inactive during the rest of the simulation. To ensure seed particle emission at a constant rate, we chose the Gaussian PIC emission model in CST with the following parameters:³⁷ (a) number of bunches = 1, (b) the inflection point for the first pulse, $\sigma = t_{\text{emission}} = 1$ ns, (c) the width of the pulse (or the cutoff length) = $t_{\text{emission}} = 1$ ns, (d) the offset between the field calculation and the particle emission = 0, and (e) the bunch distance = 0. With these specified parameters, an emission rate of 1515 particles/ns was generated by CST. We specify the number of electrons, N_e , contained in each particle at the beginning of a simulation that is kept fixed for all the particles throughout the simulation. Each of these initially injected primary particles has an initial kinetic energy, $E_{k0} = 0$ eV. We turn off space-charge effects in CST (i.e., no volume space charge and no image charge field) for results presented in Sec. IV and then turn on space-charge effects for results presented in Sec. V. With a time step of 0.01 ns, we run the simulation for $t = 20$ ns, which corresponds to 20 cycles of the fundamental excitation signal. For a specific combination of angular rf frequency, gap

distance d , and accelerating voltage V_{rf} , if we observe an exponential growth of electron population in our simulation, we conclude that multipactor discharge develops in the system for this set of parameters.

IV. RESULTS WITHOUT SPACE CHARGE EFFECTS

A. Comparing MC and CST models with analytical theory

First, we compare susceptibility boundaries obtained from analytical theory and from the MC simulation for single-frequency rf operation ($\beta = 0$). In Fig. 4(a), the MC simulation is conducted for secondary electrons with fixed emission energy, $V_0 = \frac{1}{2}mv_0^2 = 2$ eV, and emission angle $\phi = \pi/2$, which is the same as in the analytical theory. In this case, the susceptibility charts obtained from the MC simulation and the analytical calculation are in good agreement. In Fig. 4(b), the MC simulation is conducted with random emission energy and emission angle for secondary electrons given by (6). We note that allowing random emission energy and angle for secondary electron emission results in a relatively smaller region of multipactor susceptibility in Fig. 4(b) compared to that in Fig. 4(a).

We also observe from both Figs. 4(a) and 4(b) that portions of the susceptibility regions obtained from the MC simulation (red dots) lie outside the analytical susceptibility boundaries. To explain this, we study multipactor particle trajectories for a few representative cases of successful multipactor development from the MC simulation with different combinations of V_{rf} and fd . For the cases in Figs. 5(a) and 5(b) corresponding to the points m and n in the susceptibility chart in Fig. 4(b), we observe that the transit time of the macroparticle from the bottom plate to the top plate ($\tau_{A \rightarrow B}$) is equal to the transit time of the macroparticle from the top plate to the bottom plate ($\tau_{B \rightarrow A}$). For the case in Fig. 5(a), multipactor develops for an emission rf phase $\theta = 0.9468 = 0.3014\pi$, and the macroparticle crosses the gap distance in $N = 1$ half cycle of the rf period. Therefore, the corresponding point m in Fig. 4(b) lies inside the analytical susceptibility region of $N = 1$ mode. For the case in Fig. 5(b), multipactor develops for an

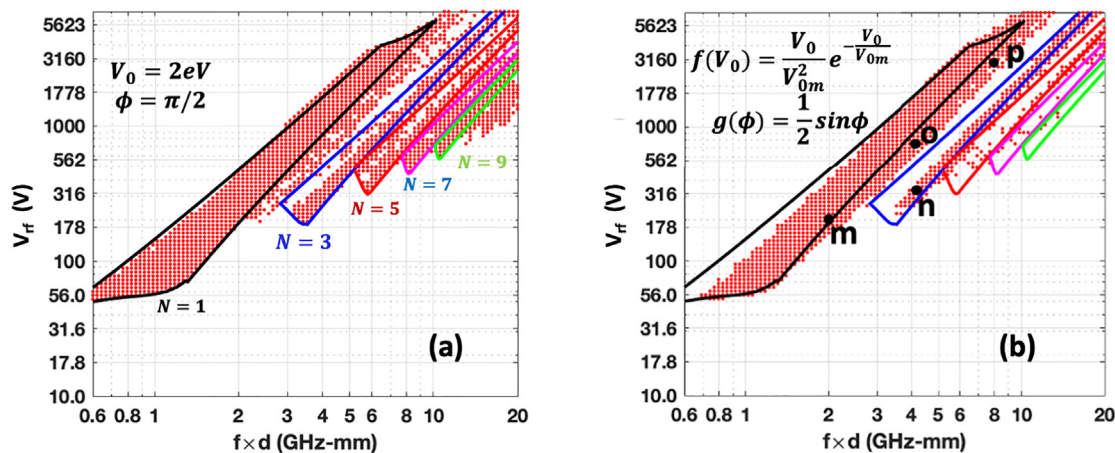


FIG. 4. (a) Multipactor susceptibility chart calculated from MC simulation (red dots) for fixed emission energy $V_0 = \frac{1}{2}mv_0^2 = 2$ eV and emission angle $\phi = \pi/2$ along with analytical boundaries for the first five modes, $N = 1$ (solid black line), 3 (solid blue line), 5 (solid red line), 7 (solid magenta line), and 9 (solid green line), calculated as described in Sec. III A. (b) Same plot as in (a) but for random emission energy and emission angle given by (6). These simulations were produced using Vaughan's SEY model parameters for copper $E_{\text{max}0} = 277.5$ eV and $\delta_{\text{max}0} = 2.088$. For all cases, we use rf frequency, $f = 1$ GHz, and variation of fd is obtained by varying the gap distance, d .

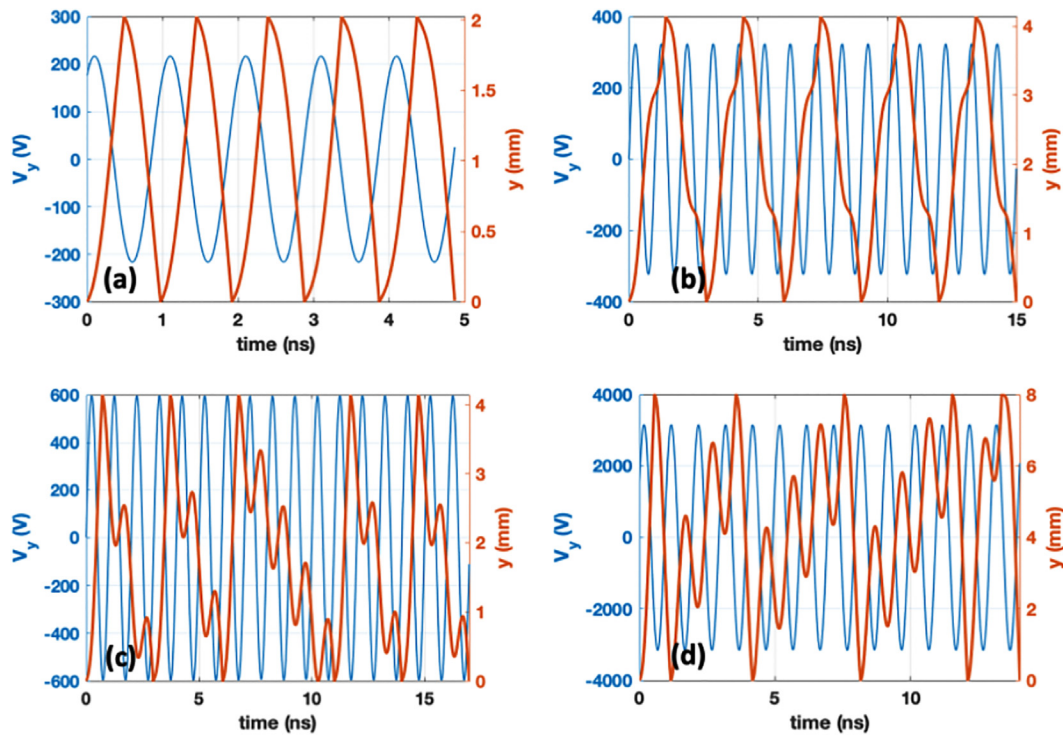


FIG. 5. Instantaneous rf voltage $V_y = V_{rf} \sin(\omega t + \theta)$ (blue lines) and the corresponding macroparticle trajectories (red lines) for (a) $V_{rf} = 216.5$ V, $fd = 2.025$ GHz mm, (b) $V_{rf} = 323.3$ V, $fd = 4.143$ GHz mm, (c) $V_{rf} = 600$ V, $fd = 4.143$ GHz mm, and (d) $V_{rf} = 3160$ V, $fd = 8$ GHz mm. For all the cases, we use rf frequency, $f = 1$ GHz, and the variation of fd is obtained by varying the gap distance, d . Plots (a)–(d) correspond to the points m, n, o, and p in the susceptibility chart in Fig. 4(b), respectively.

emission rf phase $\theta = 0$, and the macroparticle crosses the gap distance in $N = 3$ half cycles of the rf period. Therefore, the corresponding point n in Fig. 4(b) lies inside the analytical susceptibility region of $N = 3$ mode.

However, for the cases of Figs. 5(c) and 5(d) where multipactor develops for emission rf phases of $\theta = 0$ and $0.4974 (= 0.1583\pi)$, respectively, the transit times, $\tau_{A \rightarrow B} \neq \tau_{B \rightarrow A}$, and the total time required for the round trips, τ_{round} , is not always constant. Therefore, these cases give rise to mixed multipactor modes, and the corresponding points o and p in Fig. 4(b) lie outside the analytical susceptibility region. Thus, we note that due to the assumption that electrons must traverse the gap distance in odd integer multiples of the rf half cycles, the analytical model is unable to resolve these mixed multipactor modes as predicted in the MC results.

Next, we benchmark the CST simulation for single-frequency rf operation. Figure 6(a) shows the comparison of the susceptibility charts obtained from analytical calculations described in Sec. III A (solid lines), MC simulation described in Sec. III B (red dots), and CST described in Sec. III C (black dots). We observe that there is excellent agreement between the MC and the CST simulation. Typical simulation times to generate a full susceptibility diagram [black dots in Fig. 6(a)] were on the order of a few hours to a day using a PC with two 12-core Intel Xeon E5-2687WV4 processors.

For higher multipactor modes ($N = 5, 7$, and 9) in Fig. 6(a), some points in the $V_{rf} - fd$ space are found to be susceptible to multipactor through the MC simulation, while CST simulation shows that

they are not susceptible to multipactor. The difference between the MC and CST results is due to the different simulation methodologies and secondary electrons' energy distribution functions adopted in the two models. In MC, all the emitted secondary electrons are assumed to be contained in a single macroparticle, which is then emitted from the surface following the energy and angle distributions given by (6). In CST, discrete secondary particles are emitted with energies following the gamma distribution³⁷ [red curve in Fig. 6(b)], which is different from the distribution used in the MC simulation [black curve in Fig. 6(b)]. Because of the polar angle distribution of the emitted secondary particles in CST given by (7), we note a lateral diffusion of electrons in the three-dimensional CST simulation, as shown in Fig. 7.

B. Multipactor susceptibility charts for two-frequency rf operation

Figure 8 shows the multipactor susceptibility charts calculated from MC simulations for random emission energy and angle following (6), and from CST simulations for two-frequency operation with $V_{rf}[\sin(2\pi ft + \theta) + \beta \sin(n(2\pi ft + \theta) + \gamma)]$, for the cases with relative strength of the second carrier $\beta = 1$, frequency ratio between the two carrier modes $n = 2$, and relative phase of the second carrier $\gamma = 0, \pi/2$, and $3\pi/4$. It is important to note that even though the total rf power under the two-frequency operation is doubled for integer frequency separation between the carrier modes (i.e., n is integer) compared to the single-frequency operation in Fig. 8, the multipactor susceptibility region (particularly the first band) is reduced. By keeping

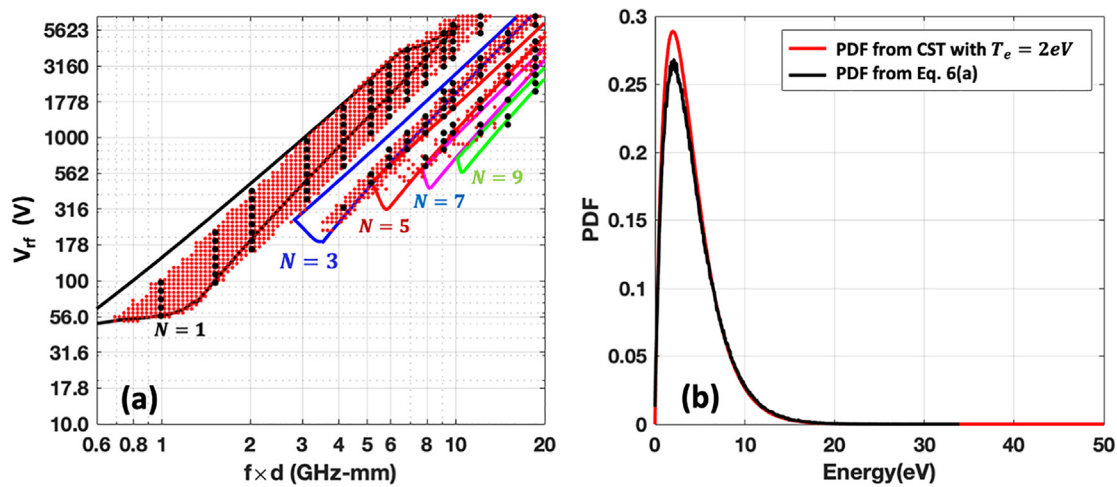


FIG. 6. (a) Comparison of the multipactor susceptibility charts obtained from analytical calculations (solid lines), MC simulation (red dots) for random emission energy and emission angle given by (6), and CST simulation (black dots). The CST simulation is conducted for $fd = 1, 1.5, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15$, and 18 GHz-mm. For all the cases, we use rf frequency, $f = 1$ GHz, and the variation of fd is obtained by varying the gap distance, d . For CST simulations, we use the number of electrons contained in each particle, $N_e = 10^5$, emission period of seed particles, $t_{emission} = 1$ ns, and the total number of seed particles, $N_{seed,total} = 1515$. (b) Emission energy probability distribution function (PDF) used in CST with $T_e = 2$ eV (red curve) and that obtained from (6a) (black curve).

the same total rf power, the multipactor susceptibility region is expected to be further reduced. From Figs. 8(b)–8(d), we observe that multipactor susceptibility regions are sensitive to the relative phase γ of the second carrier mode. As γ increases from $\gamma = 0$ to $\pi/2$, the multipactor susceptibility regions shrink; as γ further increases to $3\pi/4$, the susceptibility region increases slightly, but still smaller than the $\gamma = 0$ case. This agrees with previous results of Semenov⁴⁸ and can be attributed to the modulation of the rf envelope by the second carrier mode. We observe that in the parameter space presented in Fig. 8, the susceptibility regions have the smallest area for $\gamma = \pi/2$. The difference between the MC and the CST results can be attributed to the differences between the two methods, as described in Sec. IV A.

Figure 9 shows that as the relative strength of the second carrier mode decreases, the effect of the second carrier mode becomes less

prominent on the susceptibility chart, as the chart gradually becomes more similar to that of single frequency rf operation. A similar effect of decreasing β on the multipactor susceptibility region has been previously observed for single surface multipactor discharge with two-frequency rf fields.³¹

C. Electron trajectories at different multipactor susceptibility regions

As discussed before in Sec. III A, multipactor modes cannot be rigidly defined for two-frequency rf operation. For single frequency operation, in a single multipactor mode, as shown in Figs. 5(a) and 5(b), the transit time of the macroparticle from the bottom plate to the top plate ($\tau_{A \rightarrow B}$) and from the top plate to the bottom plate ($\tau_{B \rightarrow A}$)

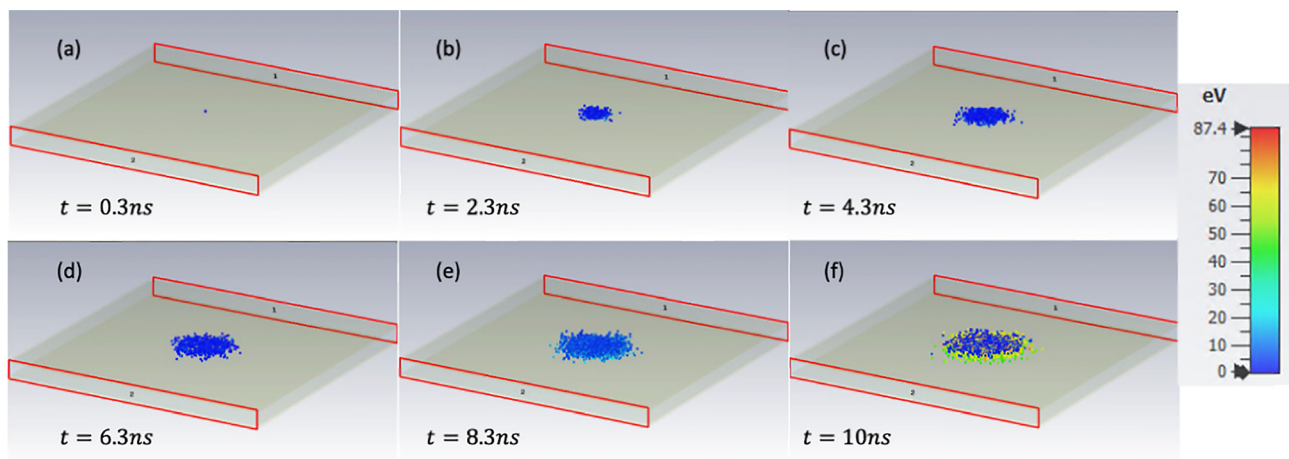


FIG. 7. Evolution of the multipactor discharge at $t =$ (a) 0.3, (b) 2.3, (c) 4.3, (d) 6.3, (e) 8.3, and (f) 10 ns, with $V_{rf} = 99$ V, $fd = 0.9902$ GHz mm, $f = 1$ GHz, and $d = 0.9902$ mm.

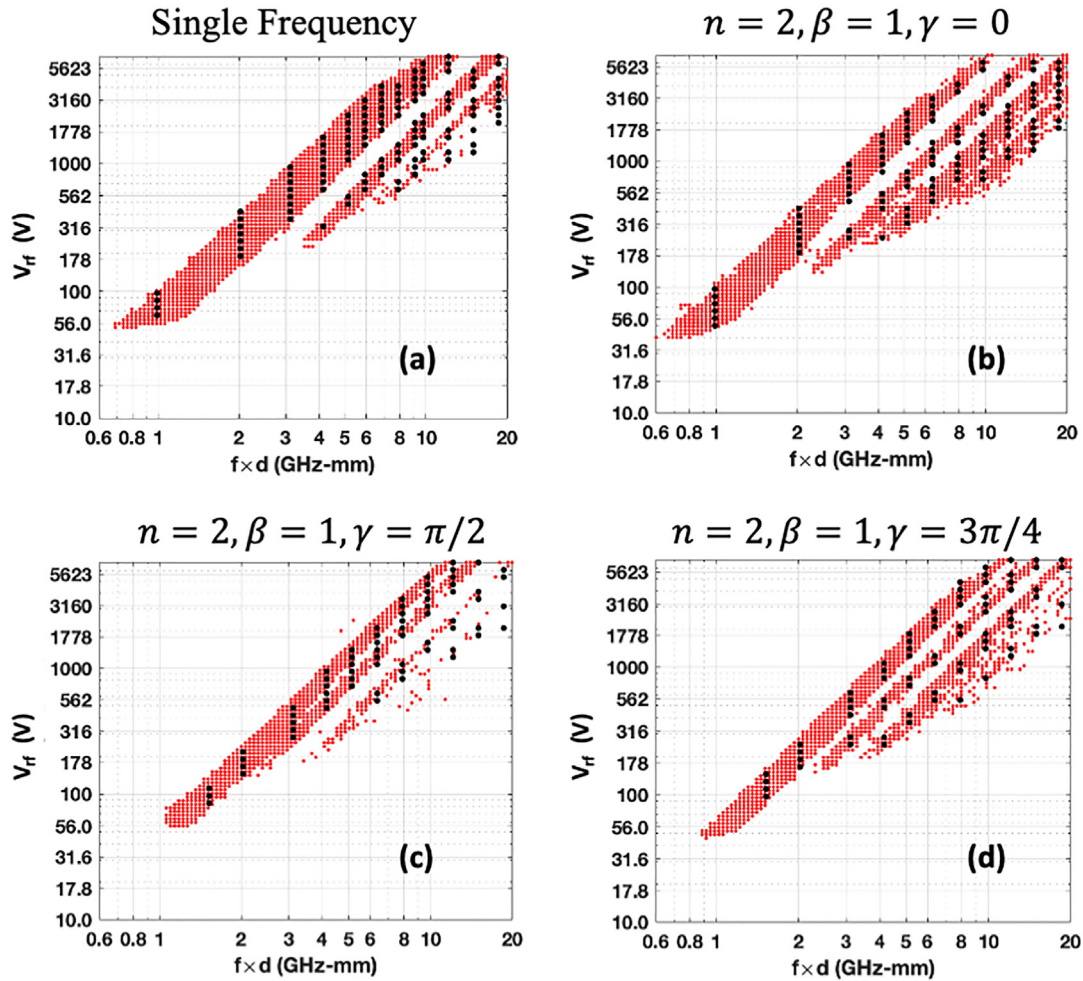


FIG. 8. Multipactor susceptibility charts in the $V_{rf} - fd$ space calculated from MC simulation (red dots) for random emission energy and emission angle given by (6) and from CST simulation (black dots) without space-charge effects for (a) single frequency rf operation, (b)–(d) two-frequency rf operations with relative strength of the second carrier $\beta = 1$, frequency ratio between the two carrier modes $n = 2$, and relative phase of the second carrier, (b) $\gamma = 0$, (c) $\gamma = \pi/2$, and (d) $\gamma = 3\pi/4$. For all the cases, we use fundamental rf frequency, $f = 1$ GHz, and the variation of fd is obtained by varying the gap distance, d . For CST simulations, we use the number of electrons contained in each particle, $N_e = 10^5$, emission period of seed particles, $t_{emission} = 1$ ns, and the total number of seed particles, $N_{seed, total} = 1515$.

are equal. However, due to the modulation of the rf envelope by the second carrier, we observe multipactor modes where $\tau_{A \rightarrow B} \neq \tau_{B \rightarrow A}$, and yet, the total time required for the complete round trip of a particle remains constant. For example, Fig. 10(b) shows a case [point m in Fig. 10(a)] of two-frequency operation with $\beta = 1$, $n = 2$, $\gamma = 0$, $V_{rf} = 56.86$ V, and $fd = 0.9902$ GHz mm, where $\tau_{A \rightarrow B} = 0.4$ ns and $\tau_{B \rightarrow A} = 0.6$ ns. The total time required for the round trip, $\tau_{round} = 1$ ns, is constant. In the case of Fig. 10(c) with $V_{rf} = 283$ V, $fd = 3.112$ GHz mm [point n in Fig. 10(a)], we have $\tau_{A \rightarrow B} = 0.8$ ns and $\tau_{B \rightarrow A} = 1.2$ ns. The total time required for the round trip is, therefore, $\tau_{round} = 2$ ns. Similarly, for the cases of Figs. 10(d) and 10(e), which are for points o and p in Fig. 10(a), respectively, we have $\tau_{A \rightarrow B} = 1.5$ and 2.5 ns, respectively, and $\tau_{B \rightarrow A} = 2.5$ and 4.5 ns, respectively. The total time required for the round trips is, therefore, $\tau_{round} = 4$ and 7 ns, respectively.

Figure 11 shows similar mixed multipactor modes for $\beta = 1$, $n = 2$, and $\gamma = \pi/2$, where $\tau_{A \rightarrow B} \neq \tau_{B \rightarrow A}$, but τ_{round} remains constant. For Figs. 11(b)–11(e), we show four cases with $\tau_{round} = 1, 1, 5$, and 7 ns, respectively.

V. SPACE-CHARGE EFFECTS

In this section, we turn on space-charge effects in CST^{37,47} and examine the effects of space-charge on multipactor susceptibility and time-dependent physics. We observe from Figs. 12(a) and 12(b) that the susceptibility charts obtained from CST (black dots) without space charge and with space charge are almost identical when we choose number of electrons per particle $N_e = 10^5$. However, if we increase the total volume charge by choosing $N_e = 5 \times 10^5$ [Fig. 12(c)], space-charge effects result in the shrinkage of multipactor susceptibility bands. This trend agrees with a previous study conducted by G.

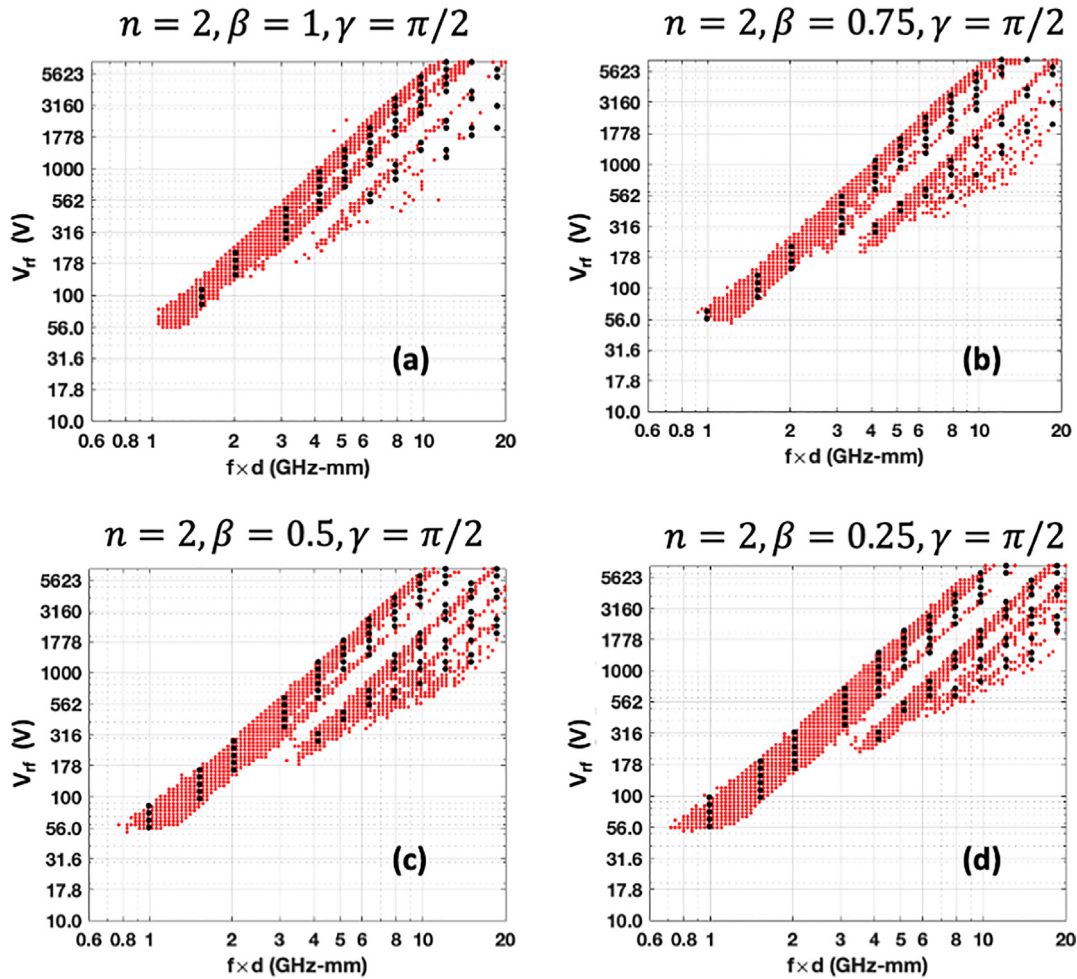


FIG. 9. Multipactor susceptibility charts in the $V_{rf} - fd$ space calculated from MC simulation (red dots) for random emission energy and emission angle given by (6) and from CST simulation (black dots) without the space-charge effect for two-frequency rf operations with frequency ratio between the two carrier modes, $n = 2$, relative phase of the second carrier, $\gamma = \pi/2$, and relative strength of the second carrier (a) $\beta = 1$, (b) $\beta = 0.75$, (c) $\beta = 0.5$, and (d) $\beta = 0.25$. For all the cases, we use fundamental rf frequency, $f = 1$ GHz, and the variation of fd is obtained by varying the gap distance, d . For CST simulations, we use the number of electrons contained in each particle, $N_e = 10^5$, emission period of seed particles, $t_{emission} = 1$ ns, and the total number of seed particles, $N_{seed,total} = 1515$.

Romanov.³⁸ Susceptibility charts with space-charge effects for different configurations of two-frequency rf operation are shown in Figs. 12(d)–12(f). Shrinkage of multipactor susceptibility bands (black dots) compared to MC calculations without space-charge effects (red dots) is prominent for each of these cases. It is noteworthy that in CST, the total volume charge and, therefore, the effects of space charge depend on the value of N_e .³² Thus, it is important to choose a proper N_e in CST simulation in order to have sufficient volume charge to observe the impact of the space charge effect.

The difference in the susceptibility charts with and without space-charge effects can be explained from the time dependent physics. As a simple representative case, we choose a point in the susceptibility chart for single-frequency rf operation ($\beta = 0$) with $V_{rf} = 85$ V and $fd = 0.9902$ GHz mm. Figure 13 shows that without space charge (blue curve), as the simulation progresses, the number of particles first increases. Then, the number of particles decreases momentarily due to

a large number of impacts with $\delta < 1$. Near the time when the electric field changes direction, i.e., $t = 1$ ns, the average secondary electron yield of impact becomes greater than unity ($\bar{\delta} > 1$), and electron population starts growing exponentially. The growth rate of the electron population without space charge (blue curve) is much higher than that with space charge (black curve).

For the CST simulation for the 1 GHz excitation, the particle source O in Fig. 3(b) emits seed particles into the vacuum gap between the plates during the first period of the excitation ($t_{emission} = 1$ ns) at a constant rate of 1515 particles/ns. Figures 14(a)–14(c) show that when space charge is absent, the number of seed particles increases linearly from $N_{seed} = 456$ to $N_{seed} = 797$ from time $t = 0.3$ to $t = 0.5$ ns according to the specified conditions in CST. These seed particles impact the bottom plate near the time $t = 0.5$ ns and the impact velocity of the primary particles $v_i > 4 \times 10^6$ m/s [Fig. 14(c)]. At $t = 0.5$ ns, the electric field reverses direction, and then the emitted secondary

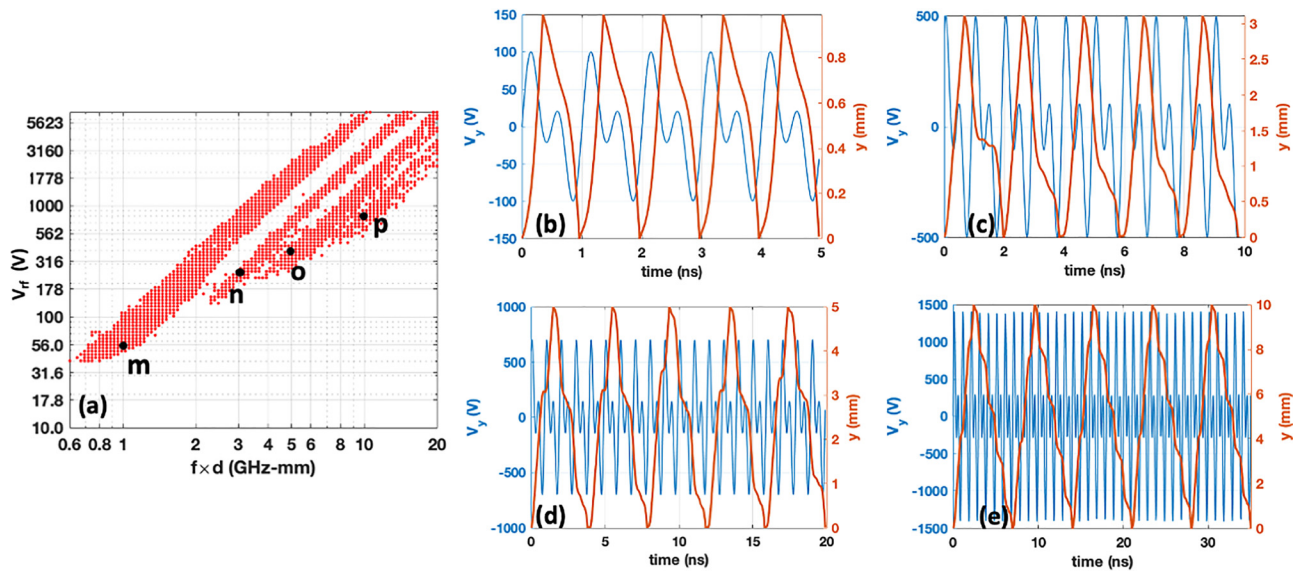


FIG. 10. (a) Multipactor susceptibility chart in the $V_{rf} - fd$ space for two-frequency rf operation with $n = 2$, $\beta = 1$, and $\gamma = 0$ obtained from MC simulation for random emission energy and emission angle given by (6). (b)–(e) Instantaneous rf voltage $V_y = V_{rf}[\sin(\omega t + \theta) + \beta \sin(n(\omega t + \theta) + \gamma)]$ (blue lines) and the corresponding macroparticle trajectories (red lines) for (b) $V_{rf} = 56.86$ V, $fd = 0.9902$ GHz mm, (c) $V_{rf} = 283$ V, $fd = 3.112$ GHz mm, (d) $V_{rf} = 400$ V, $fd = 5$ GHz mm, and (e) $V_{rf} = 800$ V, $fd = 10$ GHz mm. For all the cases, we use fundamental rf frequency, $f = 1$ GHz, and the variation of fd is obtained by varying the gap distance, d . Plots (b)–(e) correspond to the points m , n , o , and p in the susceptibility chart in (a), respectively.

particles are accelerated toward the top plate. These secondary particles impact the top plate near the time $t = 1$ ns [Figs. 14(g)–14(h)].

Electron dynamics changes significantly in the presence of space-charge effects. Figures 14(a) and 15(a) show that at time $t = 0.3$ ns,

the number of seed particles with space charge is equal to the number of seed particles without space charge ($N_{seed} = 456$ for both cases). However, at $t = 0.4$ ns, we only observe $N_{seed} = 371$ with space charge [Fig. 15(b)] compared to $N_{seed} = 607$ without space charge

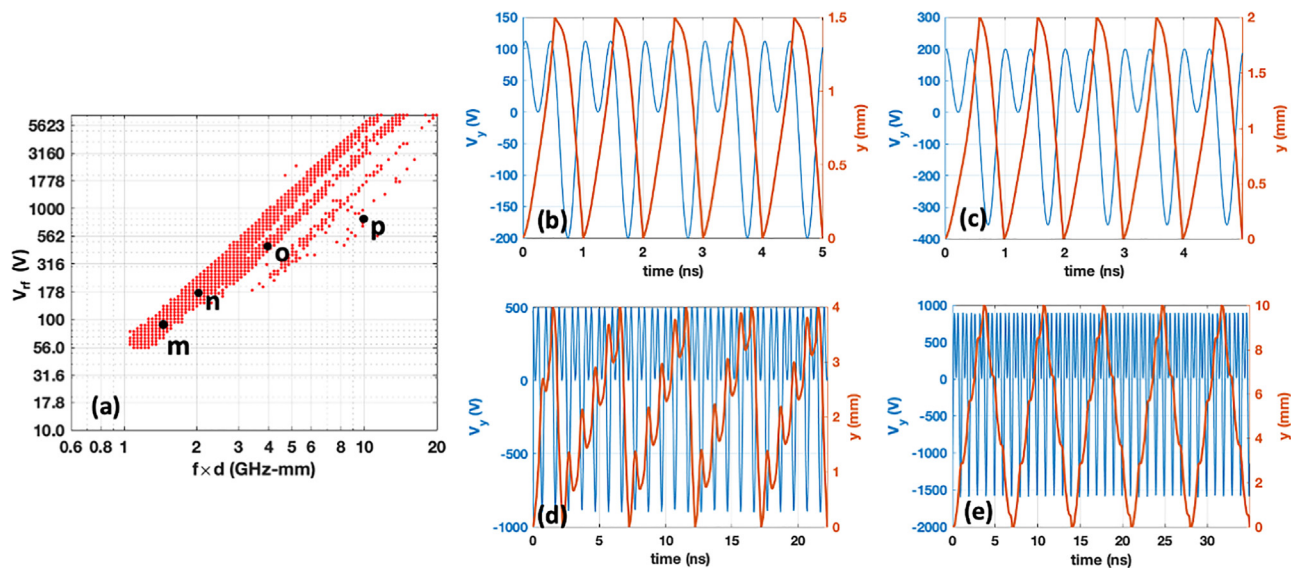


FIG. 11. (a) Multipactor susceptibility chart in the $V_{rf} - fd$ space for two-frequency rf operation with $n = 2$, $\beta = 1$, and $\gamma = \pi/2$ calculated from MC simulation for random emission energy and emission angle given by (6). (b)–(e) Instantaneous rf voltage $V_y = V_{rf}[\sin(\omega t + \theta) + \beta \sin(n(\omega t + \theta) + \gamma)]$ (blue lines) and the corresponding macroparticle trajectories (red lines) for (b) $V_{rf} = 100$ V, $fd = 1.5$ GHz mm, (c) $V_{rf} = 178$ V, $fd = 2$ GHz mm, (d) $V_{rf} = 450$ V, $fd = 4$ GHz mm, and (e) $V_{rf} = 800$ V, $fd = 10$ GHz mm. For all the cases, we use fundamental rf frequency, $f = 1$ GHz, and the variation of fd is obtained by varying the gap distance, d . Plots (b)–(e) correspond to the points m , n , o , and p in the susceptibility chart in (a), respectively.

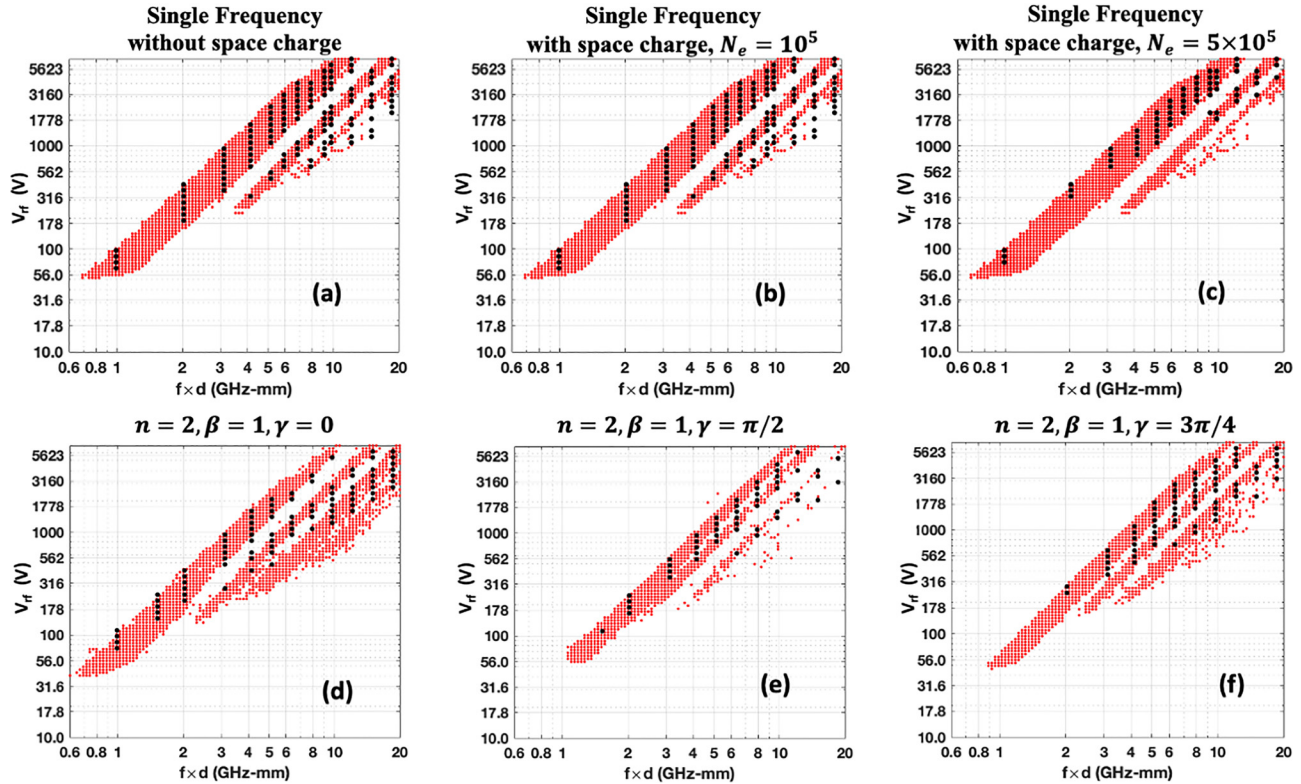


FIG. 12. Multipactor susceptibility charts in the $V_{rf} - fd$ space calculated from MC simulation (red dots) without the space-charge effect for random emission energy and emission angle given by (6) along with CST simulation (black dots). Top row: Susceptibility charts for single-frequency rf operation including CST results (a) without the space-charge effect, (b) with the space-charge effect for $N_e = 10^5$, and (c) with the space-charge effect for $N_e = 5 \times 10^5$. Bottom row: Susceptibility charts for two-frequency rf operations including CST results with the space-charge effect for $N_e = 5 \times 10^5$, $\beta = 1$, $n = 2$, and (d) $\gamma = 0$, (e) $\gamma = \pi/2$, and (f) $\gamma = 3\pi/4$. For all the cases, we use fundamental rf frequency, $f = 1$ GHz, and the variation of fd is obtained by varying the gap distance, d . For CST simulations, we use $t_{\text{emission}} = 1$ ns and $N_{\text{seed, total}} = 1515$.

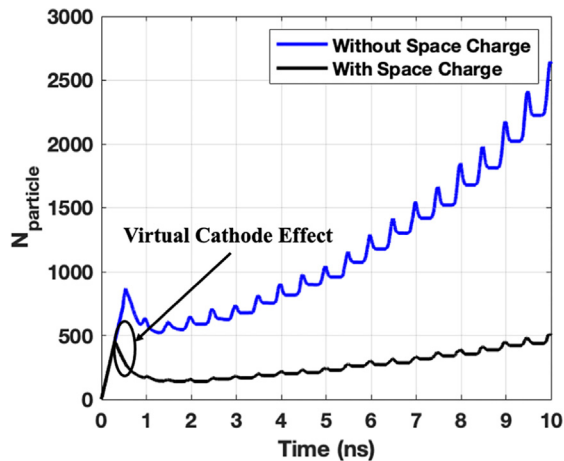


FIG. 13. Number of particles vs time in CST without space-charge effects (blue curve) and with space-charge effects (black curve). For both cases, we use $V_{rf} = 85$ V, $d = 0.9902$ mm, $f = 1$ GHz, $N_e = 10^5$, $t_{\text{emission}} = 1$ ns, and $N_{\text{seed, total}} = 1515$.

[Fig. 14(b)]. This happens because when space-charge effects are accounted for, seed particles emitted from the surface at a later time are repelled by the particles emitted during the early period and consequently absorbed by the emitting surface (top plate). This is due to the virtual cathode effect,^{40,49–52} which is also shown in Fig. 13. In addition, space-charge repulsion decreases the acceleration of the seed particles by the electric field. In fact, we observe from black circled regions in Figs. 15(b) and 15(c) that space charge causes velocity reversal of particles even before the rf field is reversed. Therefore, the velocities of the seed particles with space charge [Figs. 15(a)–15(c)] are significantly lower than those without space charge [Figs. 14(a)–14(c)]. As a result, the primary particles fail to reach the bottom plate at $t = 0.5$ ns [Fig. 15(c)] when the electric field reverses direction, i.e., the resonance condition discussed in Sec. III A is not met. The reversed electric field during $0.5 \text{ ns} < t < 1 \text{ ns}$ decelerates the seed particles approaching the bottom plate, and therefore, they impact the bottom plate with velocities $v_y < 4 \times 10^6$ m/s [Fig. 15(d)] resulting in a significantly lower SEY. In addition, many of the emitted secondary particles are absorbed by the emitting surface due to the virtual cathode effect.

Due to the virtual cathode effect induced by space charge as well as the low impact energy of primary electrons usually leading to low

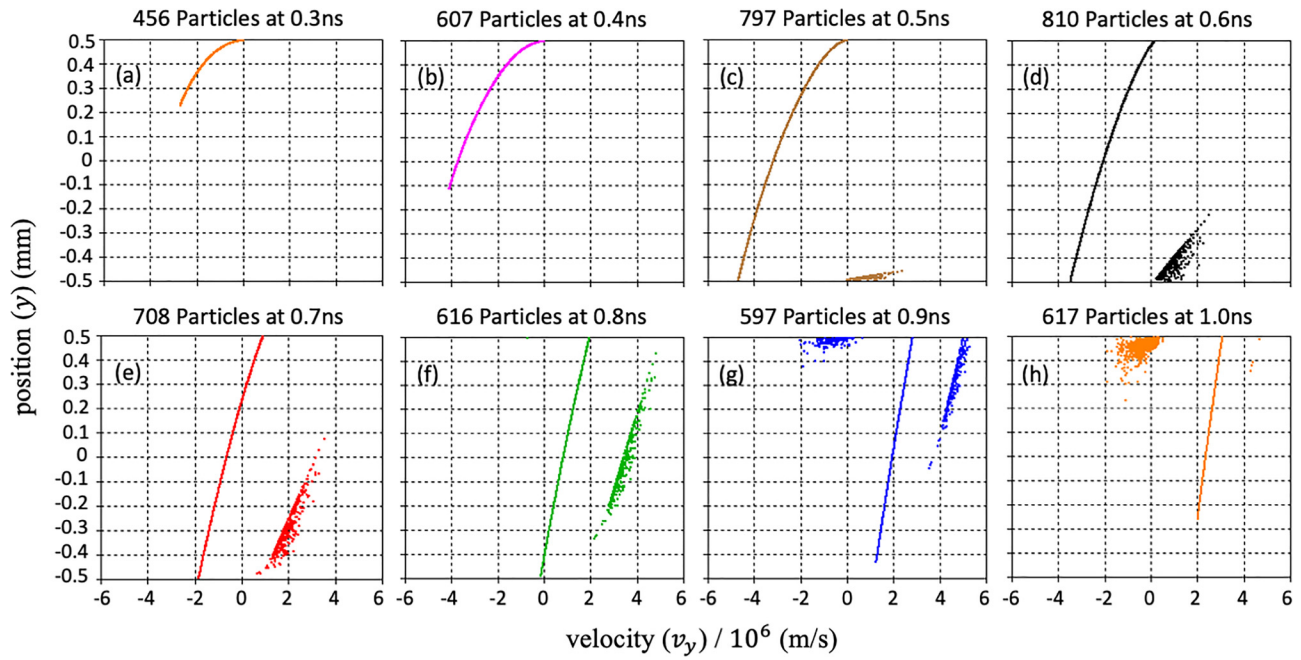


FIG. 14. y vs v_y without space-charge for $V_{rf} = 85$ V, $f = 1$ GHz, and $d = 0.9902$ mm at times (a) $t = 0.3T_{rf}$, (b) $t = 0.4T_{rf}$, (c) $t = 0.5T_{rf}$, (d) $t = 0.6T_{rf}$, (e) $t = 0.7T_{rf}$, (f) $t = 0.8T_{rf}$, (g) $t = 0.9T_{rf}$, and (h) $t = 1.0T_{rf}$, where $T_{rf} = 1$ ns is the rf period. The total number of seed particles in the simulation is set as $N_{seed, total} = 1515$, and the number of electrons contained in each particle is $N_e = 10^5$.

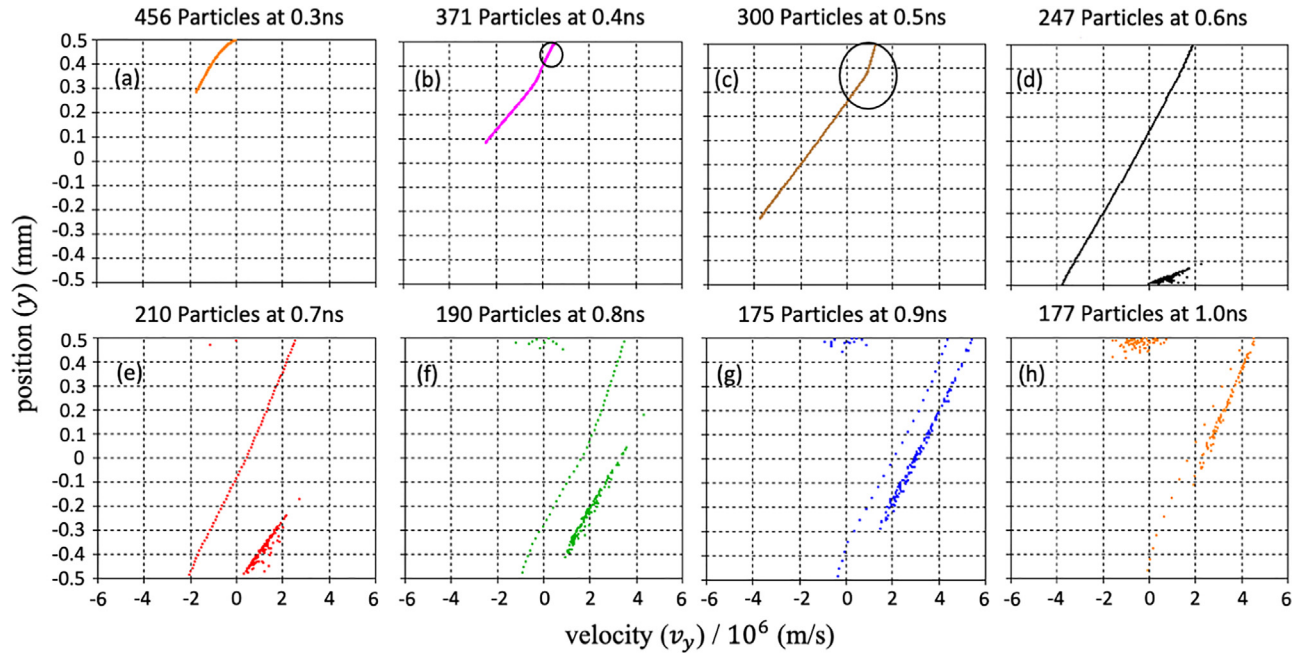


FIG. 15. y vs v_y with space-charge for $V_{rf} = 85$ V, $f = 1$ GHz, and $d = 0.9902$ mm at times (a) $t = 0.3T_{rf}$, (b) $t = 0.4T_{rf}$, (c) $t = 0.5T_{rf}$, (d) $t = 0.6T_{rf}$, (e) $t = 0.7T_{rf}$, (f) $t = 0.8T_{rf}$, (g) $t = 0.9T_{rf}$, and (h) $t = 1.0T_{rf}$, where $T_{rf} = 1$ ns is the rf period. The total number of seed particles in the simulation is set as $N_{seed, total} = 1515$, and the number of electrons contained in each particle is $N_e = 10^5$.

SEY of impacts, a higher electric field is required to initiate multipactor in the presence of space charge, which results in the upward shift of the lower multipactor susceptibility boundary. For the upper boundary, in the presence of the repulsive space-charge field, a lower RF electric field is required to make sure the emitted electrons do not back strike and get absorbed by the original emitting surface. As a result, the presence of space charge leads to the shrinkage of the susceptibility bands in Figs. 12(c)–12(f) for both single- and two-frequency operations.

VI. CONCLUDING REMARKS

We investigated two-surface multipactor with two-frequency rf fields using the Monte Carlo simulation and CST. We observed that multipactor susceptibility is sensitive to the relative strength and phase of the second carrier mode. Shrinkage of multipactor susceptibility regions is observed for different configurations of two-frequency rf operation. Regions of different multipactor modes are observed in the susceptibility chart. For multipactor modes under two-frequency operation, electrons take fixed times to complete a round trip between the two surfaces. However, the time for electrons in the forward and return trips to traverse the gap during each round trip is different.

CST simulation with space-charge effects reveals that space charge significantly changes the time dependent physics of multipactor discharge. The electron growth rate over a long period of time is lowered with space charge because of the virtual cathode effect, disruption of the resonant electron motion, and reduced SEY due to the reduced impact energy of primary electrons. Susceptibility bands in the multipactor susceptibility chart shrink in the presence of space charge.

Our study here mainly focuses on two-frequency rf operation with the second harmonic of the fundamental carrier mode. This may be of practical importance because in different rf devices such as traveling wave tubes (TWTs), a strong second harmonic component of the input rf frequency is often generated,^{53,54} and the analysis of the multipactor discharge in such practical applications requires a close examination of the effects of the second harmonic content of the fundamental rf frequency. It is also noteworthy that a sufficiently high RF field can often lead to other complicated events such as surface ablation and gas desorption. In such cases, the electron–neutral collision will create ionized plasma, which may suppress the secondary electron emission process and multipactor.⁵⁵ The presence of background gases can further assist this transition from vacuum multipactor discharge to rf plasma.⁵⁶

Future work may study the effect of non-integer frequency separation on two-surface multipactor, and the connection between single and two-surface multipactor under two-frequency operation. Two-frequency rf-induced two-surface multipactor may also be investigated in other geometries, such as microstrip, circular, and co-axial waveguides. The study may be further extended for multi-carrier rf operation, including non-sinusoidal waves. The study in this paper can also be extended to investigate multipactor induced by multi-carrier rf operation with unique geometries such as the wavy double surface,⁵⁷ or micro-structured surfaces.^{58,59} In addition, the scaling of the $V - fd$ relationship due to effects, such as varying the rf frequency beyond the electron plasma frequency, presence of magnetic fields, relativistic speeds and varying mass (e.g., ions or anions) of particles, and collision between the particles, can be explored. Correlating the macroparticle weight in CST simulation and the space charge density at ac saturation

to the space charge limited current flow might provide important insight into the scaling of the multipactor susceptibility mitigation. Future experimental efforts are also desirable to validate the theoretical and numerical results.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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